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Gears 1

Introduction to Gears

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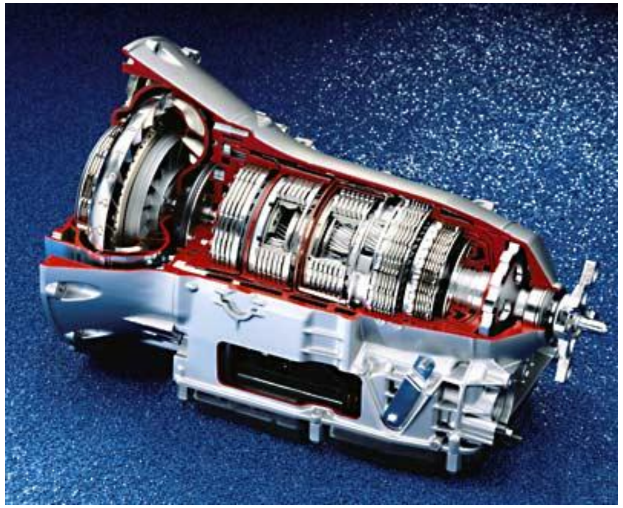
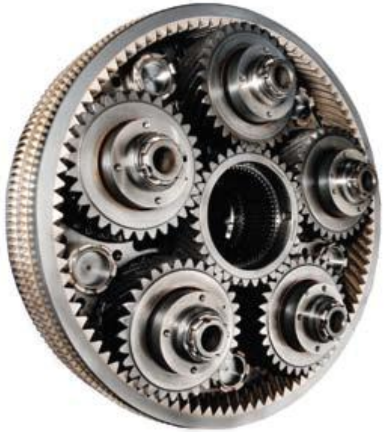
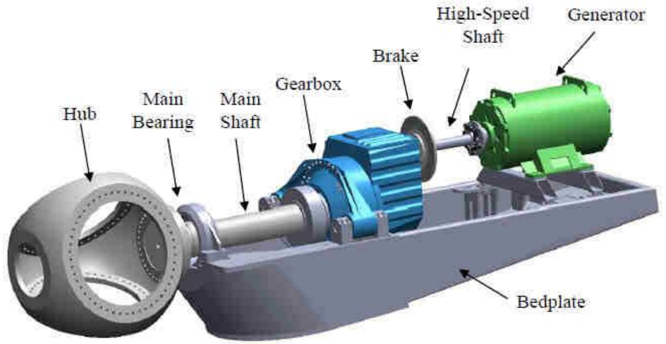
Why gears?



MB CLK, automatic transmission

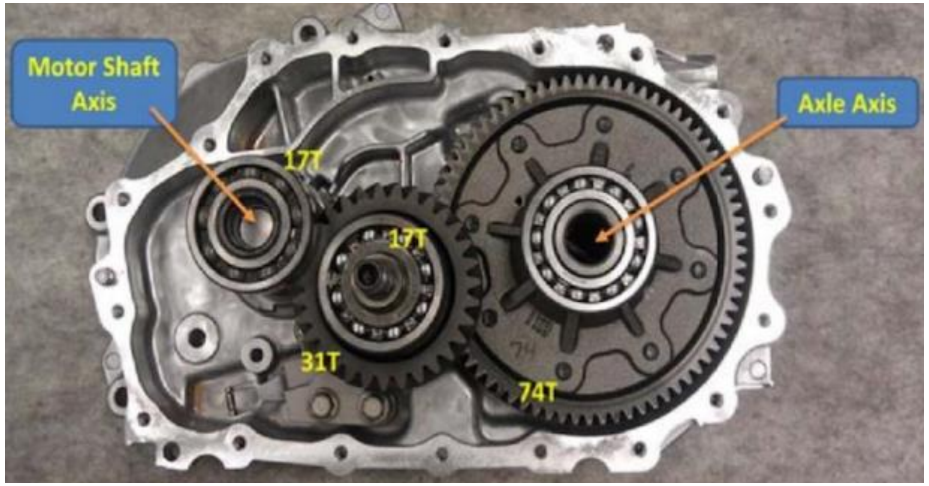


Geared turbofan for new type of aeroengine



Modular drive train configuration
http://www.nrel.gov/wind/

An earlier version of Nissan Leaf Gearbox



Automotive differential unit

Vestas V90-3MW wind turbine
http://www.vestas.com/en

Gears related Lecture sessions

Gears 1

- **Introduction to gears**
  - **Functions & types**
  - **Gear fundamentals, terminologies & conjugate action**
  - **Involute profile, fundamental equations, tooth system**

Gears 2

- **Gear trains and their applications**
  - Simple train and compound train
  - Planetary train
  - Differential unit
  - Applications

Gears 3

- **Gear stress analysis & design**
  - Common forms of gear failure
  - Gear force analysis
  - AGMA based gear design calculations

Learning objectives

- To be familiar with **different gear types** for different applications
- To understand **gear fundamentals** and to be familiar with **gear terminology**
- To be able to determine **gear ratios** and to design different **gear trains**
- To be able to determine **AGMA bending and contact stresses** of spur gears
- To understand the working mechanisms of gear systems in applications

To **understand gear fundamentals** and to be able to **design and evaluate a gear system for specific applications.**

Notes: topics on **gear manufacturing** will not be covered.

Ouline of Gears 1

- **Introduction to gears**

Part 1:
  - Functions & types of gears
  - Gear terminologies

Part 2:
  - Involute tooth profile
  - Concept of conjugate action
  - Gear ratio equation and tooth system

Part 3:
  - A few worked examples

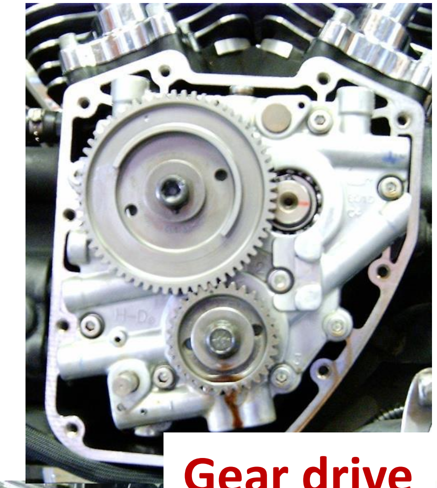
Functions and types

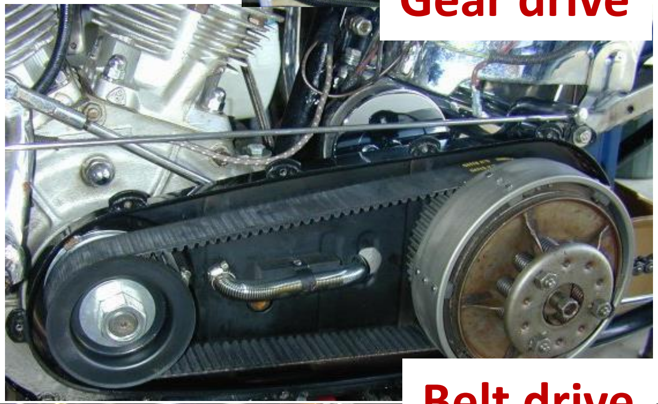
Functions:

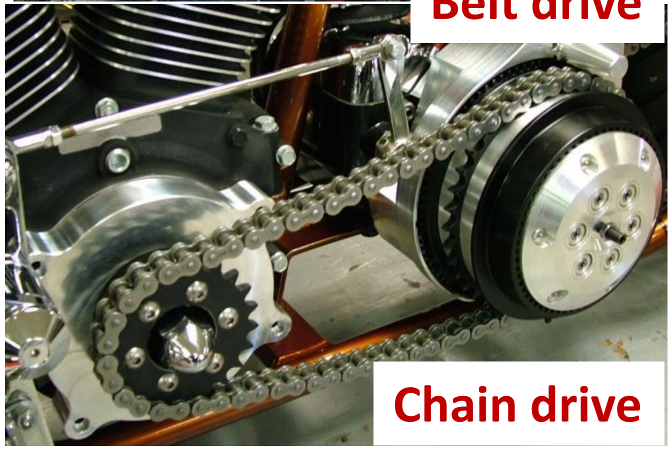
- Transmission of power or motion between two shafts
- Most rugged and durable means of power transmission
- High transmission efficiency

Classification of gear types:

- Gears between **parallel shafts**, e.g. spur & helical gears
- Gears between **intersecting shafts**, e.g. bevel gears
- Gears between **nonparallel & non-intersecting shafts**, e.g. worm gears, hypoid gears

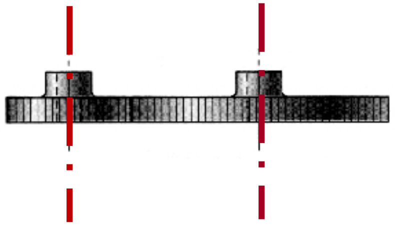

Gear drive


Belt drive

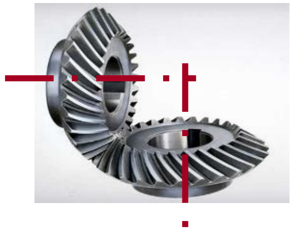

Chain drive

Gear types

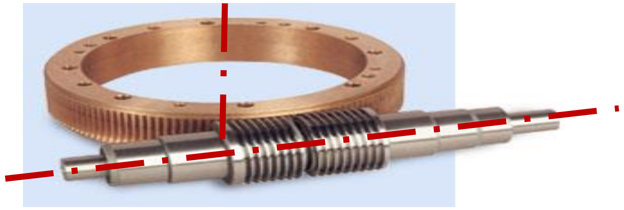
1. Parallel axis gears
   - Spur gears
   - Helical gears
   - Internal gears

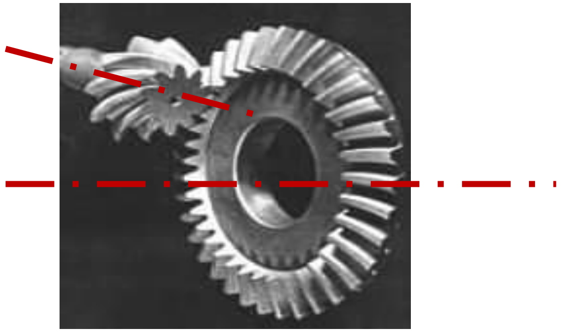


2. Non-parallel axis, coplanar gears (intersecting axes)
   - Bevel gears
   - Face gears
   - Conical involute gears



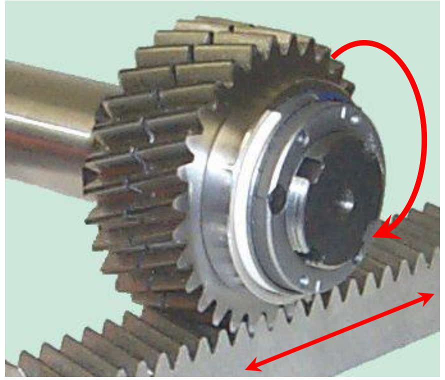
3. Non-parallel axis, non-coplanar gears (non-intersecting axes)
   - Worm gears
   - Hypoid gears



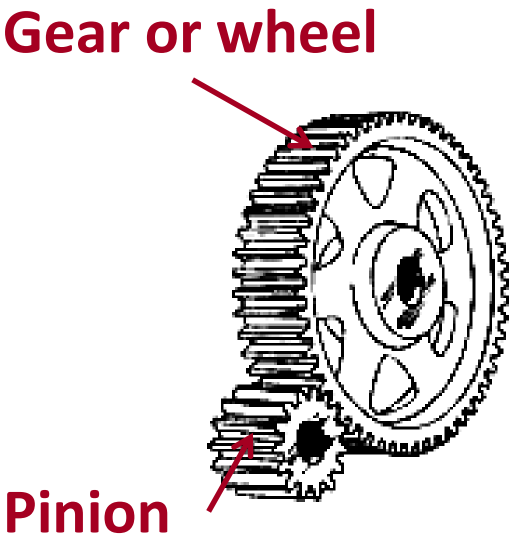


Spur, helical and herringbone gears (between parallel shafts)

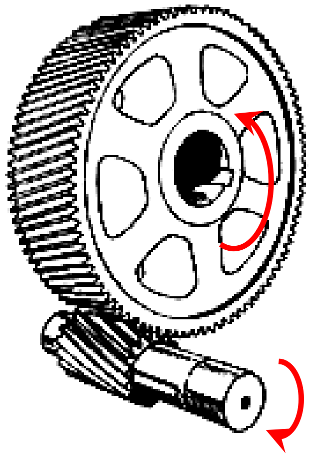
- **Spur:** tooth members parallel to shaft axes
- **Helical:** tooth members cut with an inclined angle to shaft axes, allowing smoother and quieter meshing but with an axial load generated



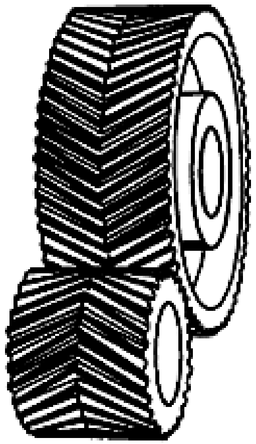
Rack and pinion



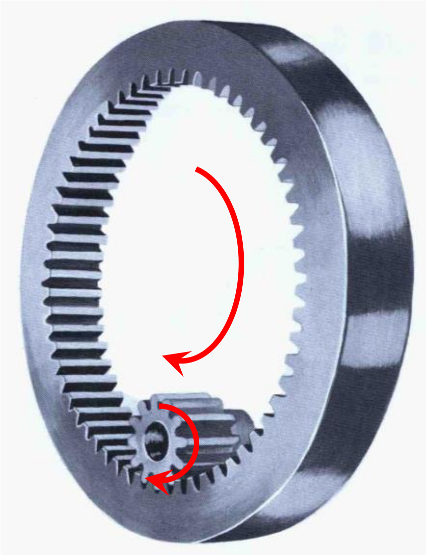
Spur gears



Helical gears



Herringbone gears (double helical gears)

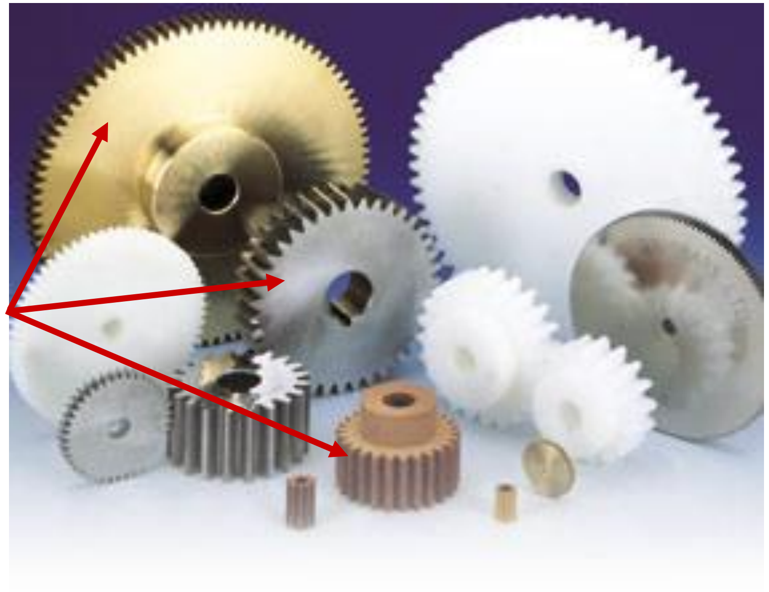


Internal gear meshing

Spur gears (between parallel shafts)

- Most common type of gears
- Cheaper to manufacture
- Generate more noise
- Smaller number of teeth in contact at any given time
- Sensitive to alignment

Different materials



Helical gears (between parallel shafts)

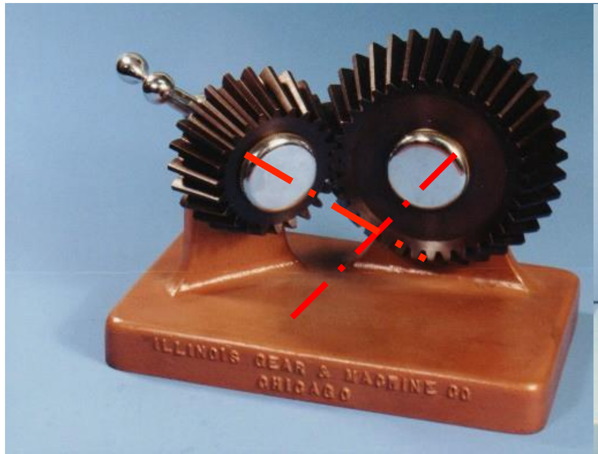
- **Teeth cut with an inclined angle to the axis of rotation**
- Contact between two teeth more progressive and longer is less noisy and carries higher loads
- Can be mounted at right angles if cut at appropriate angle
- Herringbone (double helical) gears cancel out the trust in helical gears & allow smoother power transmission at high speed

Herringbone gears

Bevel gears (between intersecting shafts)

- **Straight:** ordinary tooth profile but formed on conical surfaces, different intersecting angles may be possible
- **Spiral:** similar to helical gears with more gradual meshing

Spiral bevel gears

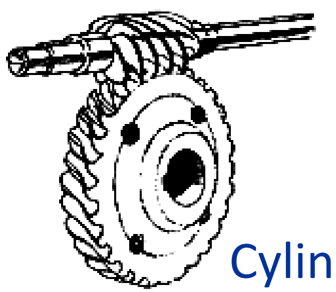


Straight bevel gears

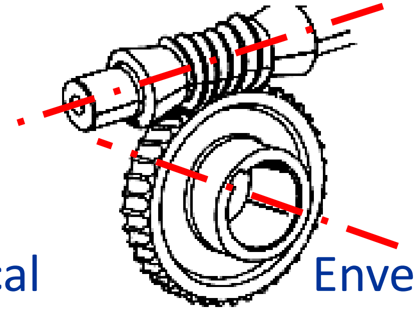
Worm gears (between non-parallel & non-interacting shafts)

Gears 1 Part 1

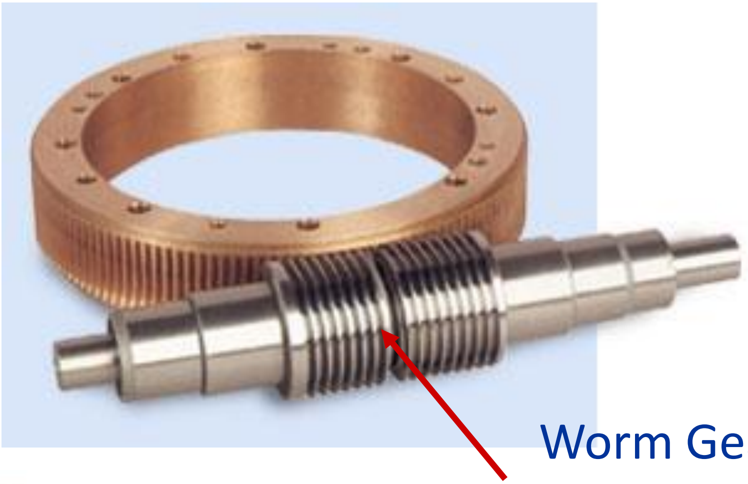
- Essentially a screw meshing with a special helical gear to provide high reduction ratio but with lower efficiency due to high sliding velocity,



Cylindrical worm gears



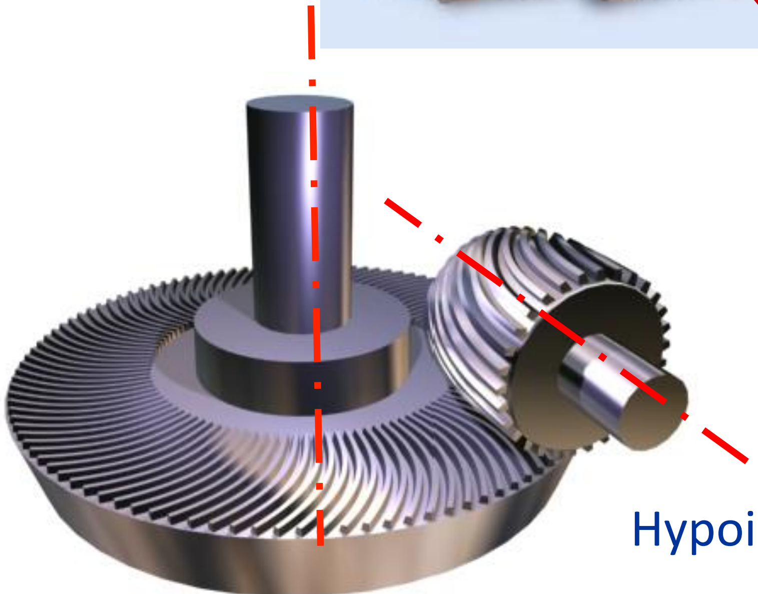
Enveloping worm gears



Worm Gear

Hypoid (bevel) gears

- Can be used between non-intersecting shafts
- Typically found in automotive differentials

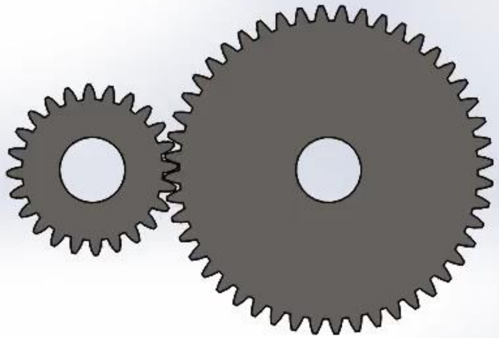


Hypoid gears

SW based animation of different gear types in operation

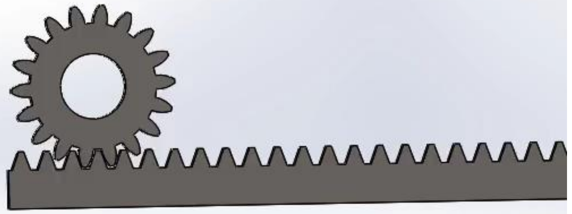
Gears 1 Part 1

Spur gears
Np=24, Ng=48

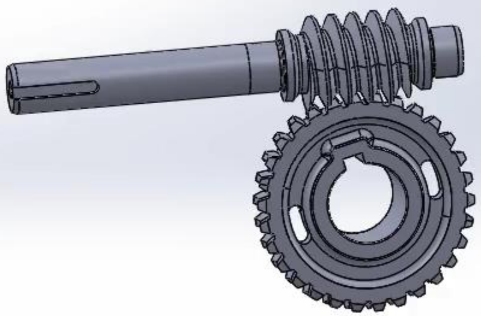


Internal spur gears
Np=18, Ng=72

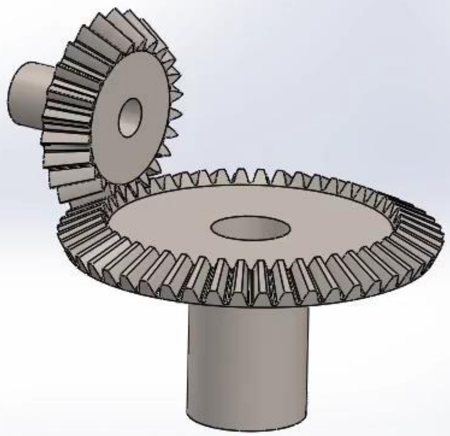
Rack & pinion
Np=20



Worm & gear
Ng=30



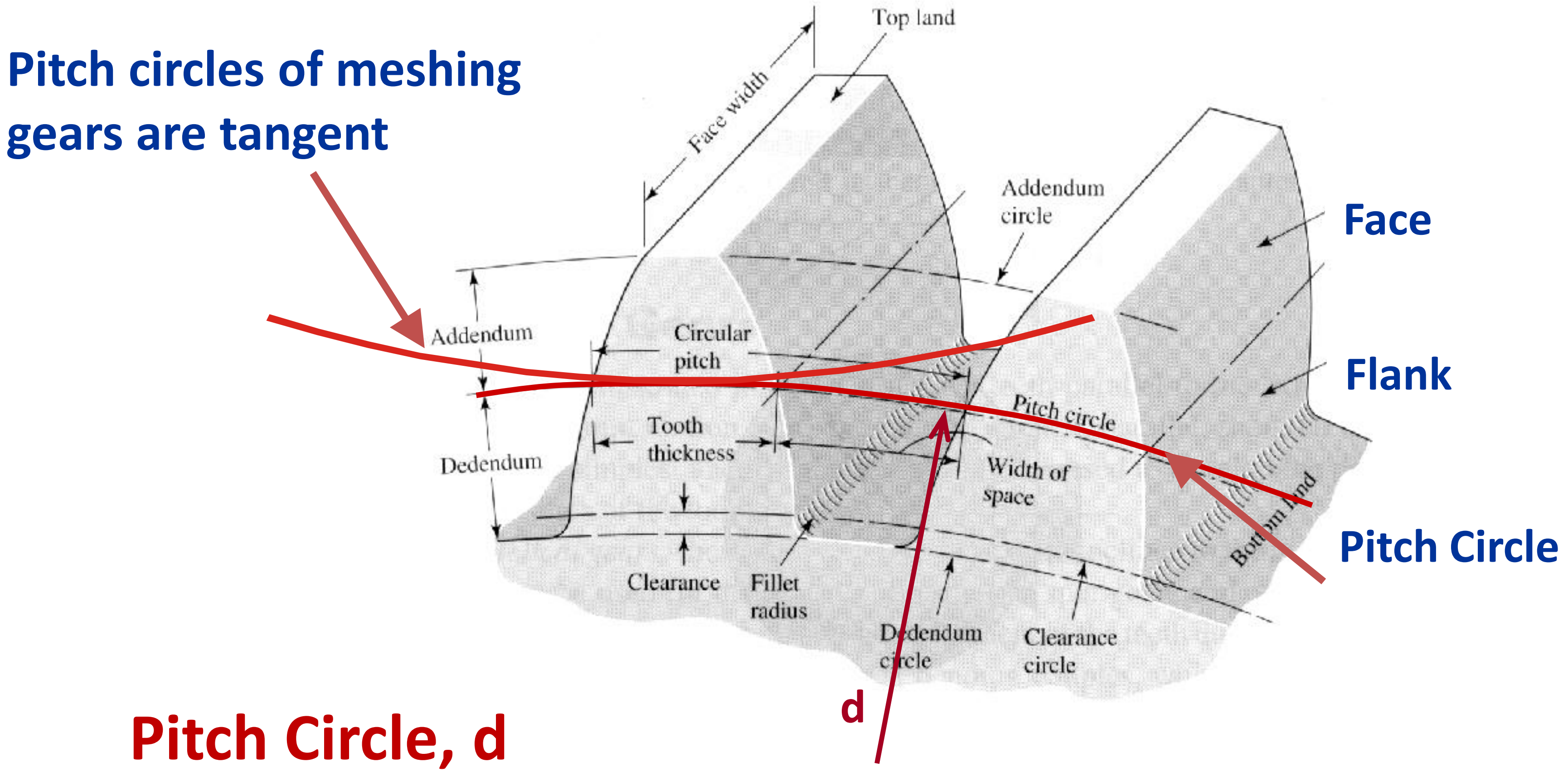
Bevel gears
Np=24, Ng=48



Note: SW assembly models of different gear types are available on Moodle

Gear terminologies

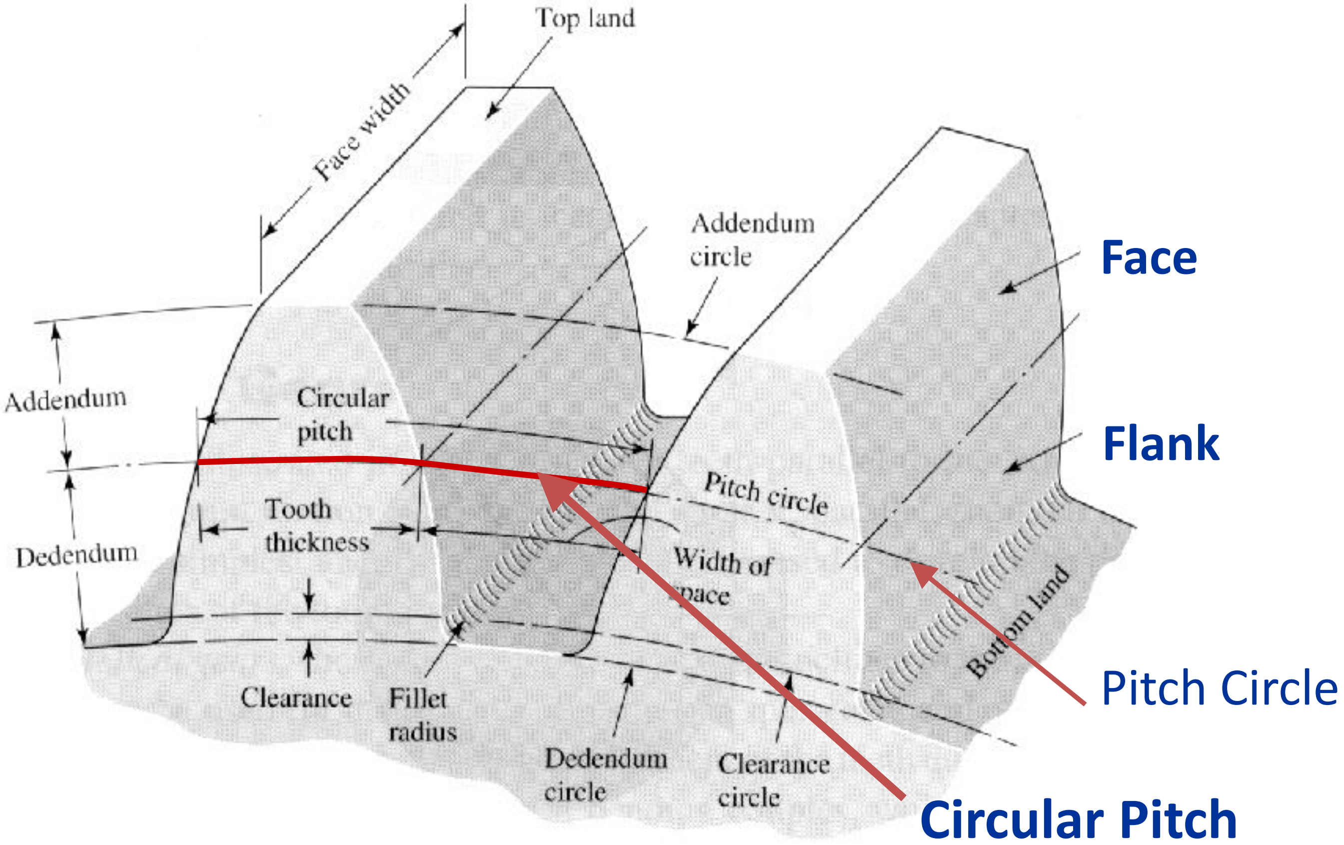
(for design evaluation, e.g. gear ratios & motion, load carrying capability & performance)



Pitch Circle, d

- Theoretical circle upon which all calculations are based
- Has a pitch diameter = ***d***

Gear terminology

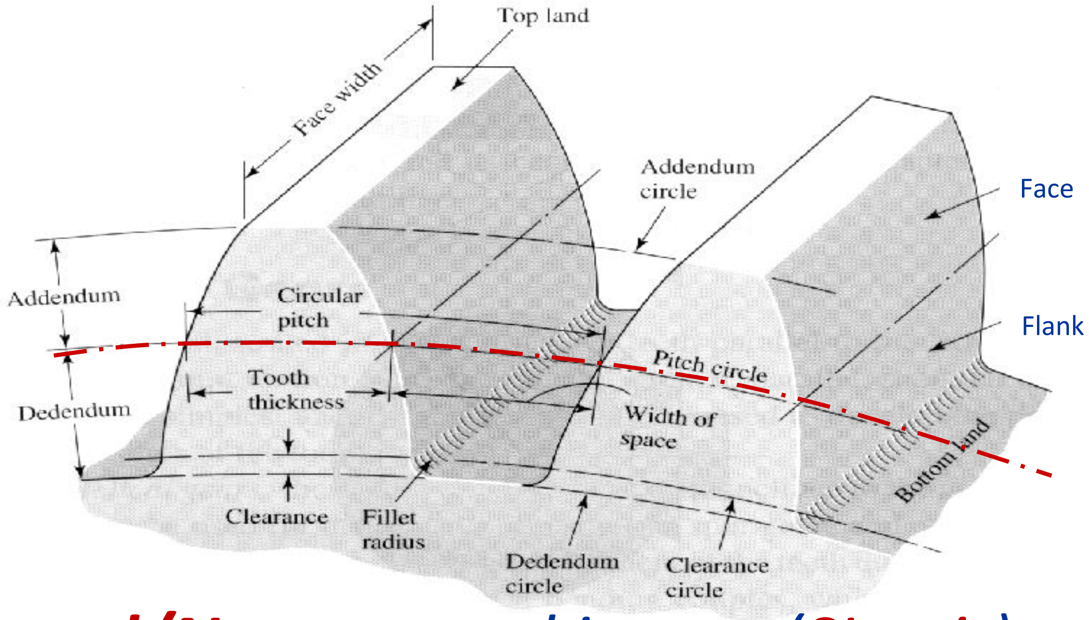


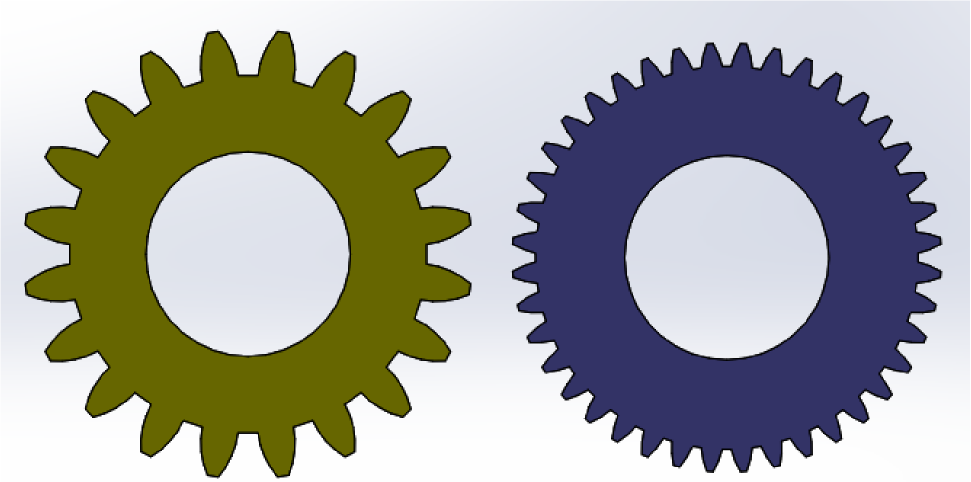
Circular Pitch, *p*=π*d*/*N*

- The distance between a point on a tooth and the same point on the adjacent tooth

- Measured on the pitch circle

Gear terminology





d = 80 mm
N = 20
m = d/N = 4 mm

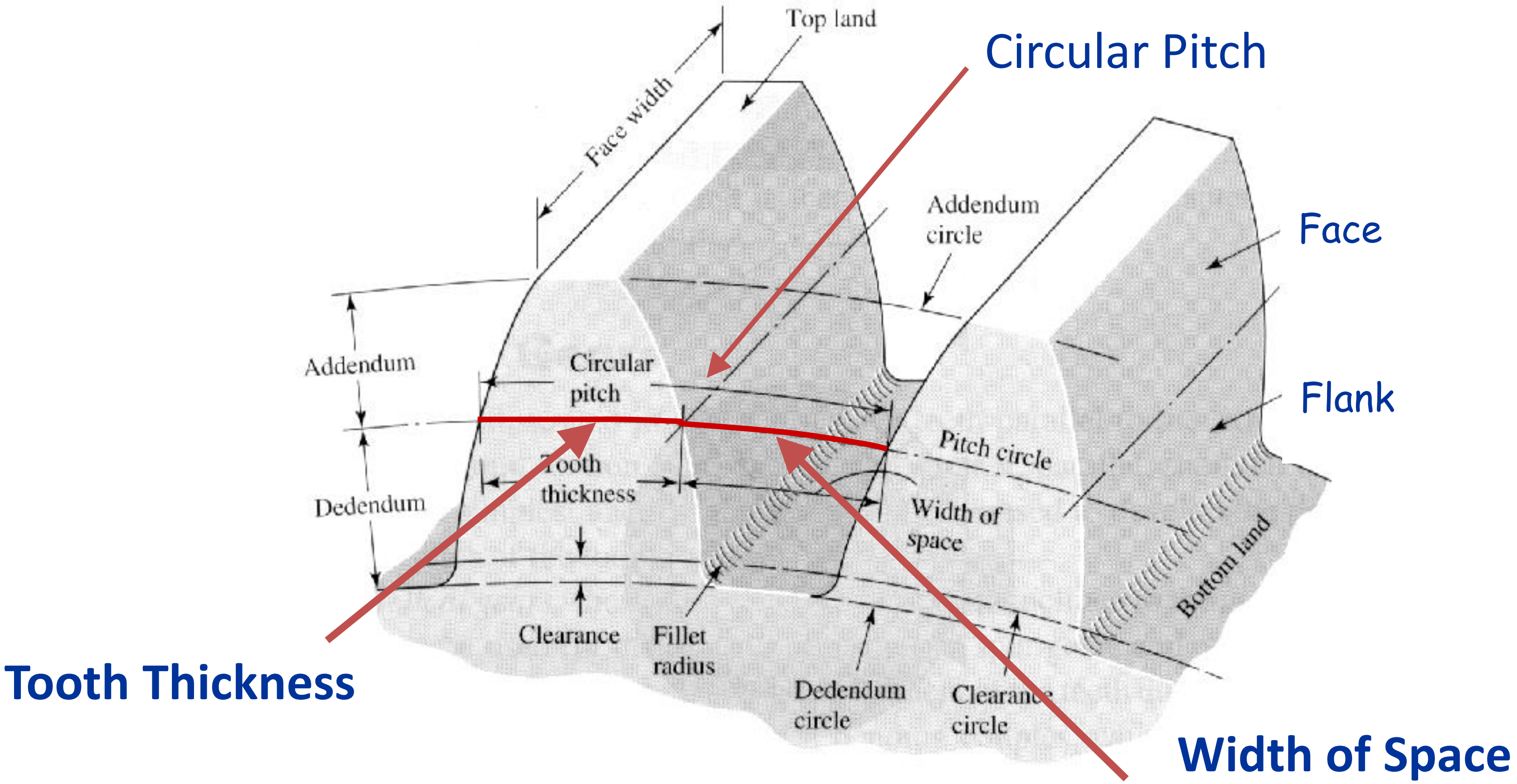
d = 80 mm
N = 40
m = d/N = 2 mm

Module, *m=d/N*, *measured in mm (SI unit)*

- The ratio of the pitch diameter ***d*** to the number of teeth ***N***
- Also circular pitch, $p = \pi d/N = \pi m$
- ***m*** represents is the **size of gear tooth** & is **standardised**

Diametral pitch, *P=N/d*, *measured in 1/in (imperial unit)*

Gear terminology



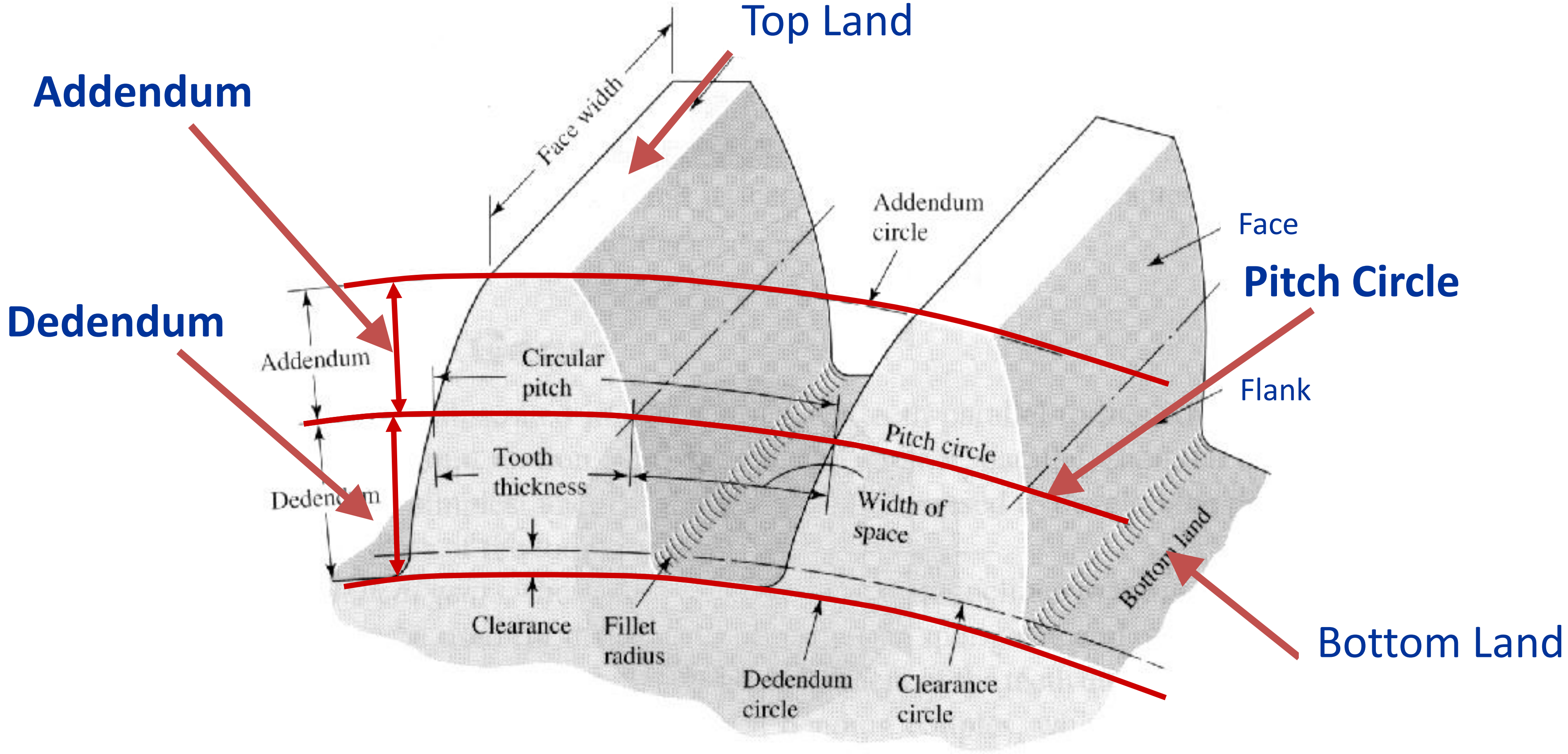
Tooth Thickness, *t=p/2=πm/2*

- Thickness of the gear tooth
- Measured on the pitch circle

Width of Space *=p/2= πm/2*

- Distance between gear teeth
- Measured on the pitch circle

Gear terminology



Addendum, ***a = m***

- Radial distance between the pitch circle and the top land

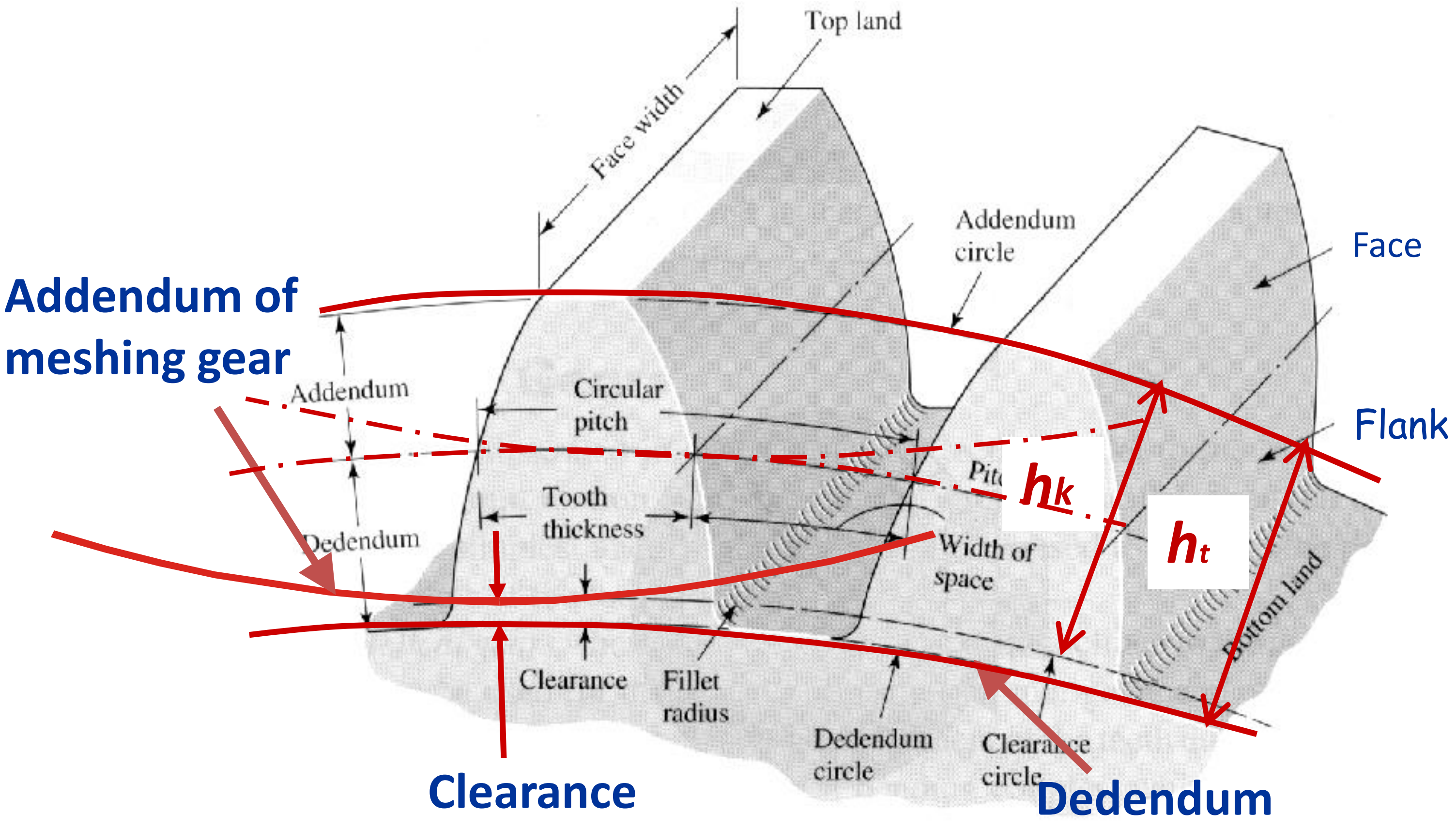
- Defined by Addendum Circle

Dedendum, *b = 1.25m*

-Radial distance between the pitch circle and the bottom land

-Defined by Dedendum Circle

Gear terminology



Clearance, ***c=0.25m (=b-a)***

- Radial distance between the dedendum circle and the addendum circle of the meshing gear

Whole depth, ***ht=a+b=2.25m***

Working depth, ***hk=a+b-c=2.0m***

Quiz 1: **True or False** to each of the following statements

Gears 1

A. **The directions of rotation of two external helical gears are opposite to each other.**

B. **Bevel gears can be used for power transmission between two non-parallel and non-intersecting shafts**

C. **Module (*m*) is a fundamental parameter to specify tooth size defined by the ratio of pitch diameter(*d*) and number of teeth (N), i.e. $m=d/N$**

D. **The Tooth thickness and Width of space are equal only on the pitch circle.**

E. **The Working depth (h_k) of a gear is the sum of Addendum (*a*) and Dedendum (*b*)**

Quiz 1: **True or False** to each of the following statements

Gears 1

A. **The directions of rotation of two external helical gears are opposite to each other.** **(true)**

B. **Bevel gears can be used for power transmission between two non-parallel and non-intersecting shafts** **(false)**

C. **Module (*m*) is a fundamental parameter to specify tooth size defined by the ratio of pitch diameter(*d*) and number of teeth (N), i.e. $m=d/N$** **(true)**

D. **Tooth thickness and Width of space are equal only on pitch circle.** **(true)**

E. **The Working depth (h_k) of a gear is the sum of Addendum (*a*) and Dedendum (*b*)** **(false)**

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Gears 1

End of Part 1

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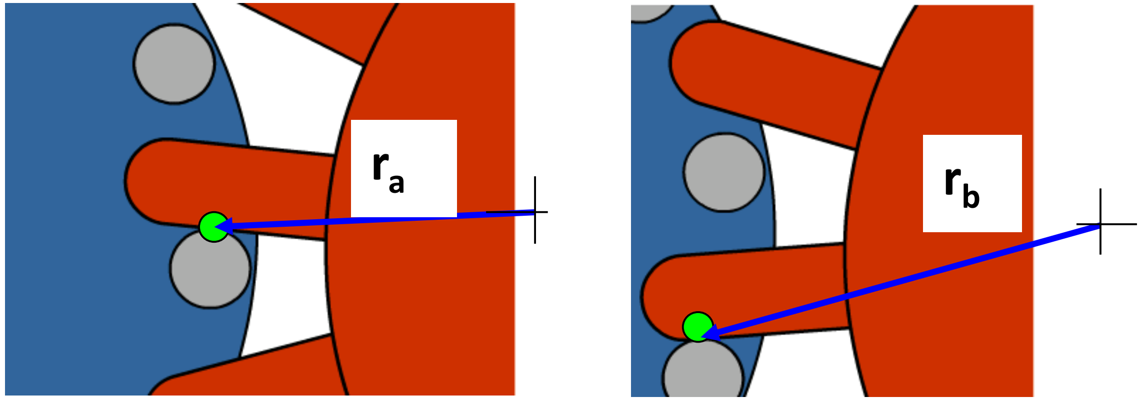
Gears 1

Part 2

Gear working mechanism: Conjugate action

Definition: When tooth profiles are designed to produce a constant angular-velocity ratio during meshing are said to have **conjugate action.**

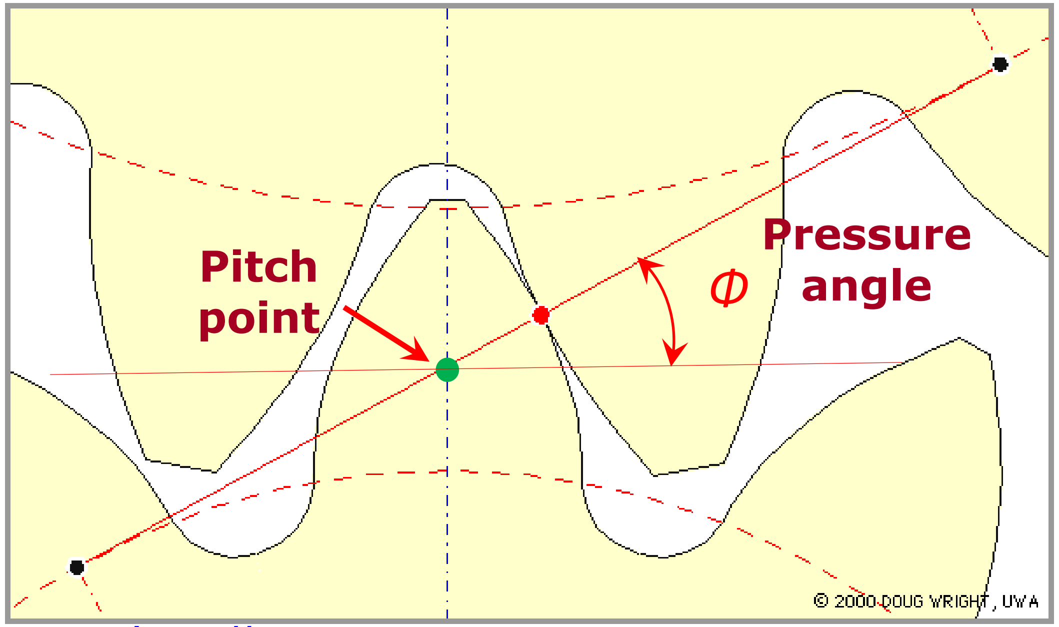
- The **common normal** to the surfaces at the point of contact always intersects the line of centres at the same **Pitch Point, P**.
- The point of contact moves along the **line of action**, tangent to the **base circles**.
- The **line of action** is always normal to the tooth profile at the point of contact.



$r_b > r_a$

Non-conjugate action of “Primitive” gears

https://www.youtube.com/watch?v=xE_l3TE-2AY

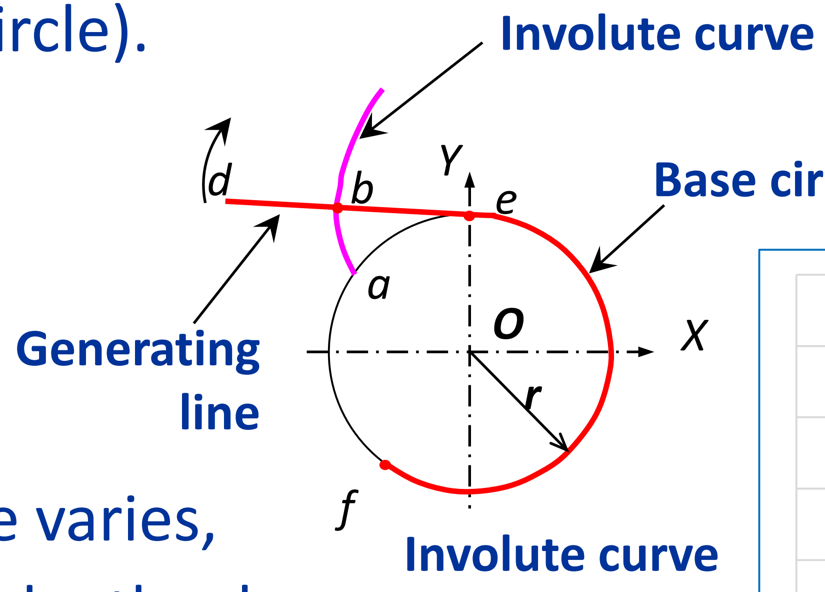


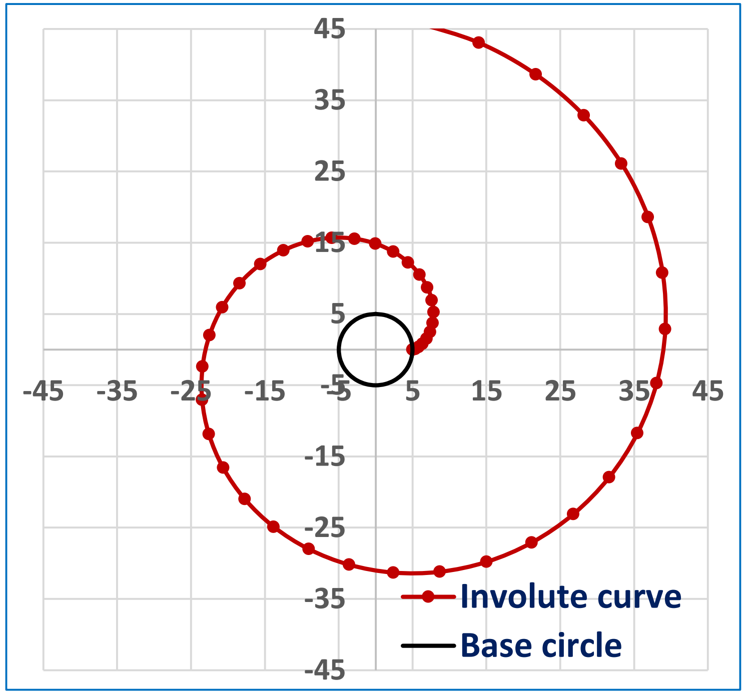
http://www.mech.uwa.edu.au/DANotes/gears/meshing/meshing.html

Gear tooth profile: Involute curve

Involute is the curve traced by a point on a tight cord as the cord is unwound from a disc or circle (base circle).

$$x = r(\cos t + t \sin t)$$
$$y = r(\sin t - t \cos t)$$





Involute properties

- Radius of involute curvature varies,
- Involute doesn't exist inside the base circle,
- **Generating line** always **normal** to the involute & tangent to the base circle,

⇩

Constant Speed Ratio

http://upload.wikimedia.org/wikipedia/commons/8/88/Animated_involute_of_circle.gif

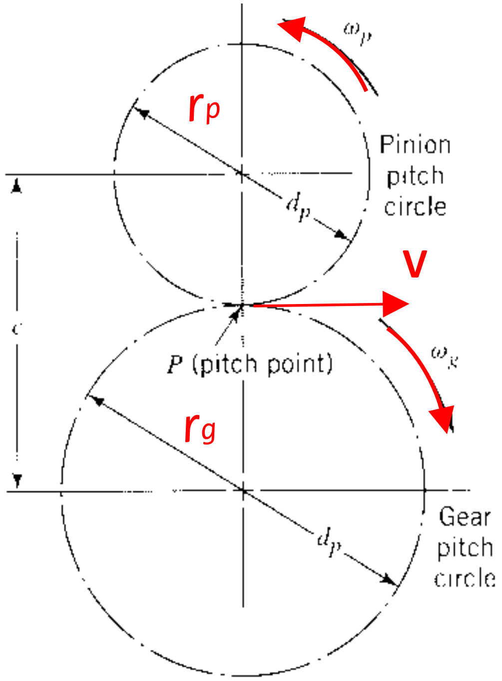
Gear ratio equation

$$V = \omega_p r_p = \omega_g r_g$$

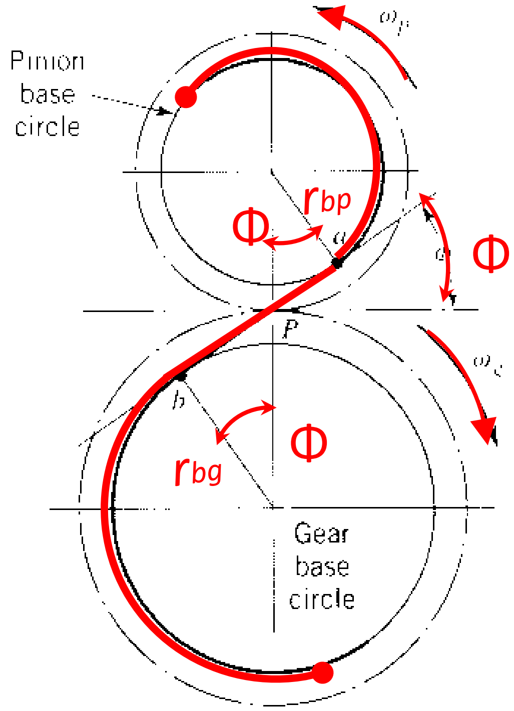
$$Z = \frac{\omega_p}{\omega_g} = \frac{r_g}{r_p} = \frac{d_g}{d_p}$$

$$\left.\begin{aligned} V_{np} &= V_{ng} \\ V_{np} &= \omega_p r_{bp} \\ V_{ng} &= \omega_g r_{bg} \\ r_b &= r\cos\phi \end{aligned}\right\} \Rightarrow Z = \frac{\omega_p}{\omega_g} = \frac{r_{bg}}{r_{bp}} = \frac{r_g \cos\phi}{r_p \cos\phi} = \frac{d_g}{d_p}$$

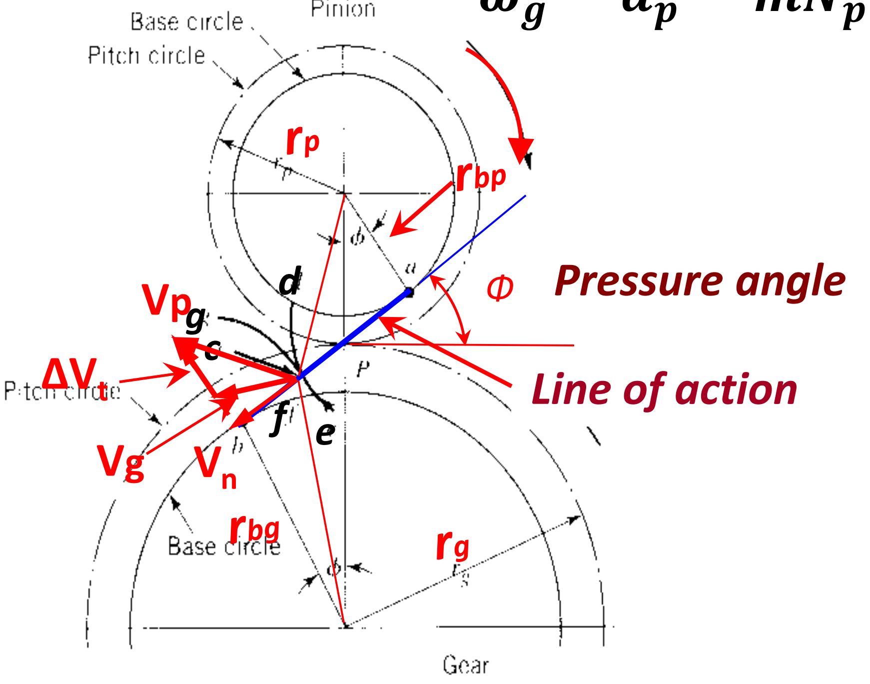
$$Z = \frac{\omega_p}{\omega_g} = \frac{d_g}{d_p} = \frac{mN_g}{mN_p} = \frac{N_g}{N_p}$$



Friction drive



Belt+friction drive



Involute gear drive

Fundamental equations

Pitch diameter: (SI unit) $$d = mN$$

Gear ratio (Z): $$Z = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{N_2}{N_1}$$

Pressure angle, ϕ, is another fundamental parameter

The necessary and sufficient conditions of proper meshing:

$$m_1 = m_2, \quad \phi_1 = \phi_2$$

where, subscripts 1 & 2 are used to denote Pinion & Gear, respectively.

Note: $\phi = 20^0$ is the most commonly used pressure angle ($\phi = 14.5^0$ is another option)

Important parameters for a pair of spur gears: $m, \emptyset \; and \; N_1, N_2$

Centre distance (C): $$C = \frac{m}{2}(N_1 + N_2)$$

Minimum number of teeth for standard gears with $\phi = 20^o$ is **18.**

Gear geometry equations

Formulas of gear geometry			
Circular pitch, p	$p = \pi m$ (mm)	Working depth, h_k	$h_k = a + b - c = 2.0m$
Pitch diameter, D	$D = mN$ (mm)	Base circle diameter, D_b	$D_b = D \cos\phi$
Addendum, a	$a = 1m$ [a)]	Outside diameter, D_o	$D_o = D + 2a$
Dedendum, b	$b = 1.25m$ [a)]	Root diameter, D_r	$D_r = D - 2b$
Clearance, c	$c = b - a = 0.25m$ [a)]	Central distance, C	$C = \frac{m}{2}(N_1 + N_2)$
Whole depth, h_t	$h_t = a + b = 2.25m$	Tooth Thickness, t	$t = \frac{\pi}{2} m$
a) Referring to standard and commonly used tooth systems for spur gears			

A simple example

For a pair of spur gears with the module of ***m=3 mm***, the numbers of teeth for the pinion and the gear are ***N1=20*** and ***N2=35***, respectively. The pressure angle is 20°. Determine the parameters of both the **pinion and gear**.

Calculation of pinion parameters *(unit in mm)*			
Pitch diameter, *D*	$D=mN1=3x20=60$	Working depth, h_k	$h_k=a+b-c=2.0m=2x3=6$
Circular pitch, *p*	$P=\pi m=3.1416x3=9.425$	Base circle diameter, D_b	$D_b=D\cos\phi=60x\cos 20^o=56.382$
Addendum, *a*	$a=1m=1x3=3$	Outside diameter, D_o	$D_o=D+2a=60+2x3=66$
Dedendum, *b*	$b=1.25m=1.25x3=3.75$	Root diameter, D_r	$D_r=D-2b=60-2x3.75=52.5$
Clearance, *c*	$c=b-a=0.25m=0.25x3=0.75$	Central distance, *C*	$C=1/2m(N_1+N_2)=0.5x3(20+35)=82.5$
Whole depth, h_t	$h_t=a+b=2.25m=2.25x3=6.75$	Tool thickness, *t*	$t=1/2\pi m=0.5x3.1416x3=4.7124$

Gear Tooth Systems

A **tooth system** is a standard to specify gear tooth geometry. The standard is used to attain interchangeability of gears of all tooth numbers of the same **Pressure Angle** (ϕ) and **Module** (m), as shown in the Table.

Key parameters of spur gears are

m, N, ϕ (either 20^o or 14.5^o)

- **Tooth sizes in General Uses (SI system)** (BS ISO 54:1996)

Module, (***m***) – SI unit (mm)	
Preferred	**0.5, 0.8, 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50**
Next choice	**1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 36, 45**

Summary

- Understand the main function and different types of gears as well as how gear systems are classified;
- Be familiar with gear terminologies and be able to determine gear ratio and key parameters with a given simple gear train (system);
- Understand the concepts of Involute profile, conjugate action, gear tooth system and fundamental requirement for proper gear meshing.

Revision questions

- How are different types of gears classified?
- Can you name a few specific forms of gears between two parallel shafts?
- How do you know the **pitch diameter (*d*)** of a gear with given **Module (*m*)** and **Number of teeth (*N*)?**
- What are the key characteristics of **conjugate action** in gear meshing?
- What are the key parameters in gear design?
- What are the **necessary and sufficient conditions** for a pair of gears meshing properly?
- Why is it useful to have **Gear tooth system** in practice?
- What is the **gear ratio equation** of a pair of gears?

Gears references and resources

- R.G. Budynas and J.K. Nisbett, Shigley's Mechanical Engineering Design, 10th ed., 2015, (TJ230 BUD)
- Childs, P.R.N., Mechanical Design Engineering Handbook, 2nd ed., 2019, (TJ230 CHI)
- http://en.wikipedia.org/wiki/Gear
- http://www.geartechnology.com
- http://powertransmission.com

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Gears 1

End of Part 2

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Gears 1

Part 3

Worked examples of Gears 1

Part 3:

- **Evaluation of simple gear train and gear parameters** as included in Gears 1 handouts available on Moodle

- **Use of SW Design Library/Toolbox and external download** from **GrabCAD,** (https://grabcad.com/) to build gear assembly models and animations

Worked example 2: Using SW Design Library/Toolbox to build Gear Assembly models

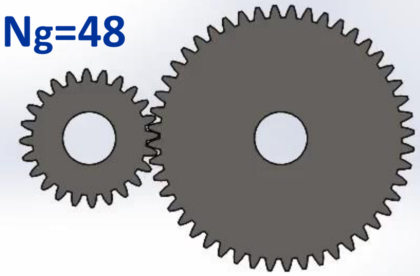
- **Solidworks Design Library/Toolbox** allows access of various types of gears
  - **SW Mechanical Mate function** allows **gear ratio based motion relations** to be defined.
  - **SW Motion Analysis** module allows simple animation and detailed motion calculations to be fully explored.

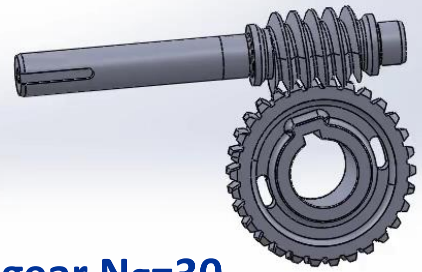
Internal spur gears

Np=18, Ng=72

Spur gears

Np=24, Ng=48





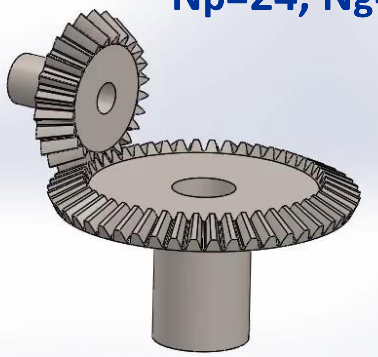
Worm & gear Ng=30

Downloaded from GrabCAD

https://grabcad.com/

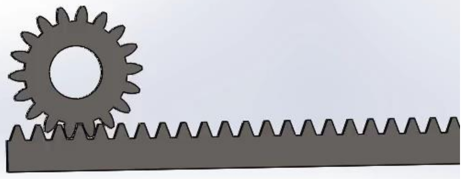
Bevel gears

Np=24, Ng=48



Rack & pinion

Np=20



SW motion analysis of gears youtube tutorial

https://www.youtube.com/watch?v=gYJVr4QuzAE

Note: SW assembly models of gears are available on Moodle

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Gears 1

End of Session